PROCEEDINGS OF SCIENCE

RBRC-778

Perturbative $O(\alpha_s a)$ matching in static heavy and domain-wall light quark system

Tomomi Ishikawa*
RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA
E-mail: tomomi@quark.phy.bnl.gov

RBC and UKQCD Collaborations

We discuss the perturbative $O(\alpha_s a)$ matching in the static heavy and domain-wall light quark system. The gluon action is the Iwasaki action and the link smearing is performed in the static heavy action. The chiral symmetry of the light quark realized by using the domain-wall fermion formulation does not prohibit the mixing of the operators at $O(a)$. The application of $O(a)$ improvement to the actual data shows that the B meson decay constant f_B, the matrix elements $\mathscr{M}_B$ and the B parameter B_B have non-negligible effects, while the effect on the SU(3) breaking ratio ξ is small.

The XXVI International Symposium on Lattice Field Theory
July 14-19 2008
Williamsburg, Virginia, USA

*Speaker.

1. Introduction

RBC/UKQCD Collaboration has been performing a large scale simulation of the lattice QCD with dynamical domain-wall fermion (DWF) [1]. In this project, we can intensively access the CKM matrix, which includes b-quark physics. To treat the b-quarks, the Heavy Quark Effective Theory (HQET) [2] is widely used. The lattice calculation with the HQET, however, has following difficulties (and solutions).

1. The static propagator is too noisy. — This is basically because the static self-energy contains $1/a$ power divergence. ALPHA collaboration investigated carefully this phenomena and introduced a modified static action which improves the signal to noise ratio [3]. The modification can be achieved by replacing the link variable in the static action with the smeared one, which is obtained by the 3-step hyper-cubic blocking. Using this the power divergence contributions in the static self-energy are largely reduced.

2. Non-perturbative matching with continuum is needed. — If we include $O(1/m_b)$ correction in the HQET formulation, the continuum limit cannot be reached by using perturbative matching factor because of power divergence [4]. Possible ways of non-perturbative matching are Schrödinger functional scheme with step scaling technique and RI/MOM scheme.

While the calculation can, in principle, be performed using the techniques described above, the actual implementation is not easy. There is an established way to apply the RI/MOM scheme for DW light quarks. But it has not been applied to the HQET successfully.

As the first step of the project, the static approximation (lowest order of the HQET) is valuable and an important approach to the complete HQET. In the static limit, the perturbative matching procedure is justified. The perturbative matching at $O(\alpha_s)$ in the static heavy and DW light quark system was calculated without link smearing in [5] and with link smearing in [6, 7]. In this report, we present the calculation including the $O(a)$ improvement, whose effect cannot be neglected in the heavy quark system that we are considering here.

2. Action setup

We use the Iwasaki gluonic action and DW fermion with light quark mass m_q for the light quark sector. The DWF has an optimized parameter M_5 which is called "domain-wall height" and takes value $0 < M_5 < 2$. In the calculation of the matching factor, it is assumed that the extension of the 5th dimension is infinity, which means the light quarks have exact chiral symmetry. For this sector, we do not carry out the link smearing.

For heavy quark sector, we use the static approximation with link smearing:

$$S_{\text{static}} = \sum_{\vec{x},t} \overline{h}(\vec{x},t) \left[h(\vec{x},t) - W_0^\dagger(\vec{x},t-1) h(\vec{x},t-1) \right], \tag{2.1}$$

where $h(\vec{x},t)$ is the effective heavy quark field and $W_0(\vec{x},t)$ is the time-component of the smeared link variable. If $W_0 = U_0$, which is the original gauge link, the action describes the one proposed

by Eichten and Hill [8]. We use the 3-step hyper-cubic blocked link for W_0 with three parameters $(\alpha_1, \alpha_2, \alpha_3)$. Possible parameter choices are

$$(\alpha_1, \alpha_2, \alpha_3) = \begin{cases} (0.0, 0.0, 0.0) & : \text{unsmeared } (W_0 = U_0) \\ (1.0, 0.0, 0.0) & : \text{APE with } \alpha = 1 \text{ [9]} \\ (0.75, 0.6, 0.3) & : \text{HYP1 [10]} \\ (1.0, 1.0, 0.5) & : \text{HYP2 [3].} \end{cases} \tag{2.2}$$

3. $O(a)$ in the static heavy and light quark system

In this report we mainly focus on the $O(a)$ improvement of operators, and then we treat the matching factor between continuum HQET (CHQET) and lattice HQET (LHQET). Perturbative matching at one-loop between continuum QCD and CHQET was obtained by Eichten and Hill [2], which we can use.

Quark bilinear operator

We consider the on-shell $O(a)$ improved static heavy (h) - light (q) quark bilinear

$$O_\Gamma^{\text{CHQET}} = Z_\Gamma (1 + b_\Gamma m_q a) \left[O_\Gamma^{(0)} + c_\Gamma a O_\Gamma^{(1)} \right], \tag{3.1}$$

relating the CHQET operator O_Γ^{CHQET} on the left hand side and LHQET operators on the right hand side. $O_\Gamma^{(0)} = \bar{h}\Gamma q$ and $O_\Gamma^{(1)} = \bar{h}\Gamma\vec{\gamma}\cdot\vec{D}q$ with $\Gamma = \{1, \gamma_\mu, \gamma_5, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$. Z_Γ is the overall matching factor between CHQET and LHQET, c_Γ and b_Γ are the $O(pa)$ and $O(m_q a)$ improvement coefficient, respectively. In this expression, we reduced the dimension 4 operators using the equation of motions of static heavy and massless light quarks

$$D_0 h = 0, \qquad \not{D} q = 0. \tag{3.2}$$

The $O(pa)$ improvement of the heavy-light currents with clover Wilson light quarks was investigated using one-loop perturbation theory in non-relativistic QCD [11] and the static approximation [12]. They showed that the $O(pa)$ effects give a large correction to the B meson decay constant f_B. In the light-light quark system, the existence of chiral symmetry guarantees the absence of $O(a)$ errors in the operators. For the case of the static heavy-light quark system, however, there are $O(a)$ effects even if we use chiral fermions for the light quarks. This was already found in the clover Wilson light quark with Wilson parameter $r = 0$ (It is chirally symmetric, but there are doublers.) [12].

Now we consider the symmetries which the theory has. In addition to the chiral symmetry in the light quark sector, we have the heavy quark symmetry $h \to e^{-i\phi_j \varepsilon_{jkl}\sigma_{kl}} h$ for the heavy quark sector. These symmetries guarantee that Z_Γ is independent on Γ [13]. And also, $c_\Gamma = Gc$, $b_\Gamma = Gb$ with $\gamma_0\Gamma\gamma_0 = G\Gamma$, in which c and b are independent on Γ.

Four-quark operator

We consider the four-quark operator ($\Delta B = 2$) which is relevant for the $B^0 - \bar{B}^0$ mixing. Its (full) QCD operator is

$$O_L^{\text{QCD}} = [\bar{b}\gamma_\mu^L q][\bar{b}\gamma_\mu^L q], \tag{3.3}$$

where $\gamma_\mu^L = \gamma_\mu P_L$ and also $\gamma_\mu^R = \gamma_\mu P_R$. At the one-loop level we need to take into account only the CHQET operator

$$O_L = 2[\overline{h}^{(+)}\gamma_\mu^L q][\overline{h}^{(-)}\gamma_\mu^L q], \tag{3.4}$$

for matching between the CHQET and LHQET operators, where $h^{(+)}(h^{(-)})$ is the particle (anti-particle) of the static quark. With the use of chiral fermions for the light quark, the on-shell $O(a)$ improved four-fermion operator can be written in

$$O_L^{\mathrm{CHQET}} = Z_L\left[O_L + Z_L^{(1)} a O_{ND} + Z_L^{(m)} m_q a O_N\right], \tag{3.5}$$

where

$$\begin{aligned} O_{ND} =\ & 2[\overline{h}^{(+)}\gamma_\mu^L q][\overline{h}^{(-)}\gamma_\mu^R(\gamma_i\vec{D}_i)q] + 4[\overline{h}^{(+)}P_L q][\overline{h}^{(-)}P_R(\gamma_i\vec{D}_i)q] \\ & +2[\overline{h}^{(+)}\gamma_\mu^R(\gamma_i\vec{D}_i)q][\overline{h}^{(-)}\gamma_\mu^L q] + 4[\overline{h}^{(+)}P_R(\gamma_i\vec{D}_i)q][\overline{h}^{(-)}P_L q], \end{aligned} \tag{3.6}$$

$$\begin{aligned} O_N =\ & 2[\overline{h}^{(+)}\gamma_\mu^L q][\overline{h}^{(-)}\gamma_\mu^R q] + 4[\overline{h}^{(+)}P_L q][\overline{h}^{(-)}P_R q] \\ & +2[\overline{h}^{(+)}\gamma_\mu^R q][\overline{h}^{(-)}\gamma_\mu^L q] + 4[\overline{h}^{(+)}P_R q][\overline{h}^{(-)}P_L q], \end{aligned} \tag{3.7}$$

Z_L is an overall matching factor, $Z_L^{(1)}$ is the $O(pa)$ improvement coefficient and $Z_L^{(m)}$ is the $O(m_q a)$ improvement coefficient.

4. One-loop perturbative calculation of the $O(a)$ coefficients

Quark bilinear operator

We calculate the matching factor and the $O(a)$ improvement coefficients using one-loop perturbation theory. The calculation is performed by comparing the light to heavy scattering amplitude between the CHEQT and LHQET. Now we consider the scattering amplitude with an initial light quark q carrying momentum p and a final heavy quark h carrying momentum k. In order to extract the on-shell $O(a)$ coefficients, the amplitude is expanded in the external quark momenta p and k around zero momentum and the light quark mass m_q around zero mass. Since the momenta obey the equation of motions (3.2), $\not{p} = 0$ and $k_0 = 0$. In the puerturbative calculation, we choose the Feynman gauge and the UV divergences in the continuum calculation are regulated by dimensional regularization and we use the $\overline{\mathrm{MS}}$ scheme for the renormalization. The IR divergences are regulated by introducing the gluon mass λ.

The renormalized scattering amplitude for the CHQET at one-loop order can be written in the form

$$\begin{aligned} \langle h(k)|O_\Gamma|q(p)\rangle_{\mathrm{cont}} =\ & \left(1 + \frac{\alpha_s}{4\pi}C_F\mathscr{A}_{\mathrm{cont}}^{(0)}\right)\langle O_\Gamma^{(0)}\rangle_0 + \frac{\alpha_s}{4\pi}C_F\mathscr{A}_{\mathrm{cont}}^{(1)} a\langle O_\Gamma^{(1)}\rangle_0 \\ & + \frac{\alpha_s}{4\pi}C_F\mathscr{A}_{\mathrm{cont}}^{(m)} m_q a\langle O_\Gamma^{(0)}\rangle_0, \end{aligned} \tag{4.1}$$

where $\langle\ \rangle_0$ represents the tree level expectation value of the amplitude, $C_F = (N_c^2-1)/(2N_c)$ with number of color N_c, and

$$\mathscr{A}_{\mathrm{cont}}^{(0)} = -\frac{3}{2}\ln\left(\frac{\lambda^2}{\mu^2}\right) + \frac{5}{4}, \quad \mathscr{A}_{\mathrm{cont}}^{(1)} = -\frac{8\pi}{3a\lambda}, \quad \mathscr{A}_{\mathrm{cont}}^{(m)} = -\frac{4\pi}{3a\lambda}. \tag{4.2}$$

	unsmeared	APE	HYP1	HYP2
$\delta\hat{M}$	12.979	5.514	4.910	3.671
e	14.884	1.429	0.667	-3.378
$e_R = e - \delta\hat{M}$	1.906	-4.085	-4.243	-7.049

Table 1: Numerical values of $\delta\hat{M}$, e and e_R for each link smearing.

In this expression μ is the renormalization scale parameter. The scattering amplitude for the LHQET has the same IR divergence as in the continuum:

$$\langle h(k)|J_\Gamma|q(p)\rangle_{\rm latt} \;=\; \left(1+\frac{\alpha_s}{4\pi}C_F\mathscr{A}^{(0)}_{\rm latt}\right)\langle J^{(0)}_\Gamma\rangle_0 + \frac{\alpha_s}{4\pi}C_F\mathscr{A}^{(1)}_{\rm latt}\,a\langle J^{(1)}_\Gamma\rangle_0 + \frac{\alpha_s}{4\pi}C_F\mathscr{A}^{(m)}_{\rm latt}(1-w_0^2)m_q a\langle J^{(0)}_\Gamma\rangle_0, \qquad (4.3)$$

where $w_0 = 1 - M_5$,

$$\mathscr{A}^{(0)}_{\rm latt} = -\frac{3}{2}\ln\left(a^2\lambda^2\right) + \frac{f+e_R}{2} + d^{(0)}, \quad \mathscr{A}^{(1)}_{\rm latt} = -\frac{8\pi}{3a\lambda} + d^{(1)}, \quad \mathscr{A}^{(m)}_{\rm latt} = -\frac{4\pi}{3a\lambda} + d^{(m)}. \quad (4.4)$$

The value of f was obtained in [14]. Since we will use the fitting function $\sim e^{-Et}$, $e_R = e - \delta\hat{M}$, the reduced value of e, is used [8]. The values are presented in Tab. 1. $d^{(0)}$, $d^{(1)}$ and $d^{(m)}$ are the finite parts of the vertex correction whose values are shown in Fig. 1. After the matching we obtain the renormalized operator with $O(a)$ improvement

$$O^{\rm CHQET}_\Gamma \;=\; (1-w_0^2)^{-1/2}Z_w^{-1/2}Z_\Gamma\left(1+b_\Gamma(1-w_0^2)m_q a\right)\left[O^{(0)}_\Gamma + c_\Gamma a O^{(1)}_\Gamma\right], \qquad (4.5)$$

where

$$Z_\Gamma \;=\; 1+\frac{\alpha_s}{4\pi}C_F\left[\frac{3}{2}\ln\left(a^2\mu^2\right)+\frac{5}{4}-\frac{f+e_R}{2}-d^{(0)}\right], \qquad (4.6)$$

$$c_\Gamma \;=\; -\frac{\alpha_s}{4\pi}C_F G d^{(1)}, \quad b_\Gamma = -\frac{\alpha_s}{4\pi}C_F G d^{(m)}. \qquad (4.7)$$

Because of our use of DW light quarks we need the DW-specific factors $(1-w_0^2) = (1-(1-M_5)^2)$ and $Z_w = 1+\frac{\alpha_s}{4\pi}C_F z_w$ [14] in Eq. (4.5). The $O(\alpha_s a)$ coefficients Eq. (4.7) are new results of this calculation.

Here we should briefly mention the $1/a$ power divergence in the operator $O^{(1)}_\Gamma$ caused by the mixing with the lower dimensional operator $O^{(0)}_\Gamma$. Since this power divergence is already in the $O(a)$ part, the total contribution is $O(a^0)$ and we do not worry about it in taking the continuum limit. And also this $O(a^0)$ effect contributes at $O(\alpha_s^2)$, which we can neglect in this one-loop calculation. This is quite different from the power divergence that appears in the $1/m_b$ expansion: if the matching is done at l-th loop, the power divergence of $\sim \alpha_s^{l+1}/a$ remains.

Four-quark operator

The calculation for the four-quark operator can be done just by rearranging the quark bilinear results above. After the matching we obtain the $O(a)$ improved operator

$$O^{\rm CHQET}_L \;=\; (1-(w_0)^2)^{-1}(Z_w)^{-1}Z_L\left[O_L + Z^{(1)}_L a O_{ND} + Z^{(m)}_L(1-w_0^2)m_q a O_N\right]. \qquad (4.8)$$

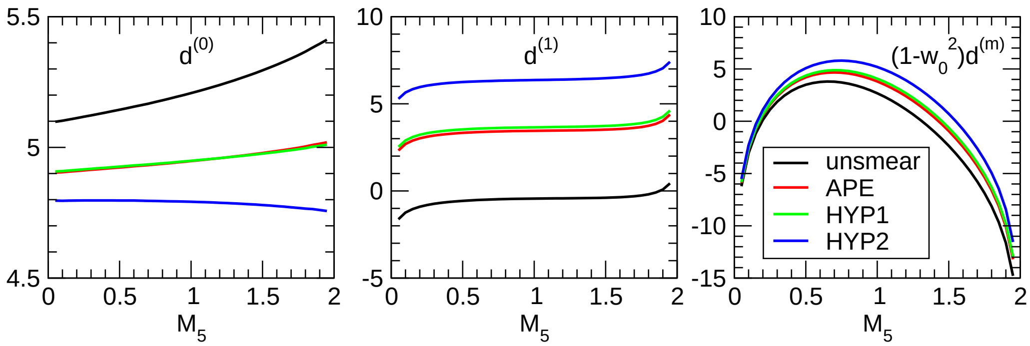

Figure 1: Numerical values of $d^{(0)}$, $d^{(1)}$ and $(1-w_0^2)d^{(m)}$ versus DW height M_5 for each link smearing.

As in the quark bilinear operator, we need the DW-specific factors in Eq. (4.8). The matching factor and $O(a)$ coefficients are

$$Z_L^{(0)} = 1+\frac{\alpha_s}{4\pi}\left[4\ln\left(a^2\mu^2\right)+\frac{7}{3}-\frac{10}{3}d^{(0)}-\frac{c}{3}-\frac{v}{3}-\frac{4}{3}e_R-\frac{4}{3}f\right], \tag{4.9}$$

$$Z_L^{(1)} = \frac{\alpha_s}{4\pi}\cdot 2d^{(1)}, \quad Z_L^{(m)} = \frac{\alpha_s}{4\pi}\cdot 2d^{(m)}, \tag{4.10}$$

where the constant v is the one-loop contribution from the diagram in which the gluon connects light and light, which was obtained in [14]. The constant c arises when the gluon connects two heavy lines and is given by $c = e_R$.

5. Discussion

In Eqs. (3.1) and (3.5), we used the operators $O_\Gamma^{(1)}$ and O_{ND} which contain covariant derivatives. These operators, however, can be written in the form:

$$\overline{h}^{(\pm)}\Gamma\vec{\gamma}\cdot\vec{D}q = \mp G\partial_0\left(\overline{h}^{(\pm)}\Gamma q\right), \quad O_{ND} = 2[\overline{h}^{(+)}\gamma_\mu^L q]\left(\overleftarrow{\partial}_0-\overrightarrow{\partial}_0\right)[\overline{h}^{(-)}\gamma_\mu^L q], \tag{5.1}$$

where we have used the equations of motion (3.2). This form is quite convenient for taking the $O(a)$ improvement in correlation functions. In the evaluation of the 2-point correlation function $\langle A_0^{(-)\mathrm{imp}}(t)A_0^{(-)\dagger}(0)\rangle$, where $A_0^{(-)} = \overline{h}^{(-)}\gamma_0\gamma_5 q$, we have

$$\langle A_0^{(-)\mathrm{imp}}(t)A_0^{(-)\dagger}(0)\rangle = \left(1+b_\Gamma(1-w_0^2)m_q a+c_A aE_{\mathrm{bind}}\right)\langle A_0^{(-)}(t)A_0^{(-)\dagger}(0)\rangle. \tag{5.2}$$

E_{bind} is the binding energy of static heavy and light quark, which is obtained in the correlator fitting. Therefore, in order to accomplish the $O(a)$ improvement, no further measurement is needed in the 2-point correlation function. And also because the $O(m_q a)$ part can be neglected due to its small size in many cases, we omit the $O(m_q a)$ in the following discussion. Using the Eq. (5.2), we can evaluate the $O(a)$ improvement of B meson decay constant f_B like

$$f_B^{\mathrm{imp}} = (1+c_A aE_{\mathrm{bind}})\,f_B. \tag{5.3}$$

For matrix element $\mathscr{M}_B$, B parameter B_B and SU(3) breaking ratio ξ we use the vacuum saturate approximation (VSA), and we obtain

$$\mathscr{M}_B^{\mathrm{imp}} \xrightarrow{\mathrm{VSA}} \left(1+2Z_L^{(1)}aE_{\mathrm{bind}}\right)\mathscr{M}_B, \qquad B_B^{\mathrm{imp}} \xrightarrow{\mathrm{VSA}} \left(1+2(Z_L^{(1)}-c_A)aE_{\mathrm{bind}}\right)B_B,$$
$$\xi^{\mathrm{imp}} \xrightarrow{\mathrm{VSA}} \left(1+Z_L^{(1)}a(E_{\mathrm{bind}(B_s)}-E_{\mathrm{bind}(B_d)})\right)\xi. \tag{5.4}$$

Now we roughly estimate these $O(\alpha_s a)$ effect using the actual simulation data ($\beta = 2.13$, $L^3 \times T \times L_5 = 16^3 \times 32 \times 16$, $M_5 = 1.80$, $m_{ud}a = \{0.01, 0.02, 0.03\}$, $m_s a = 0.0359$) which appeared in [6]. For this estimate the MF-improvement is taken into account. In this case, $d^{(1)} = 3.48(\mathrm{APE}),\ 6.41(\mathrm{HYP2})$ and $E_{\mathrm{bind}} \sim 0.6(\mathrm{APE}),\ 0.5(\mathrm{HYP2})$. The coupling constant has the range $\alpha_s \sim 0.15-0.35$, conservatively. The conclusion is that the $O(\alpha_s a)$ effect of f_B is $3-8\%(\mathrm{APE}),\ 5-12\%(\mathrm{HYP2})$, of $\mathscr{M}_B$ is $9-24\%(\mathrm{APE}),\ 15-36\%(\mathrm{HYP2})$, and of B_B is $3-8\%(\mathrm{APE}),\ 5-12\%(\mathrm{HYP2})$. Using the assumption $(E_{\mathrm{bind}(B_s)} - E_{\mathrm{bind}(B_d)}) \sim (m_{B_s} - m_{B_d})$, the effect for ξ is less than 2%.

6. Summary

We have presented a one-loop perturbative calculation of the $O(a)$ improvement coefficient for the static heavy - DW light quark system taking into account the link smearing in the heavy quark sector. Estimated $O(a)$ effect is not small in f_B, $\mathscr{M}_B$ and B_B, but is small in ξ. While perturbative matching has large ambiguities and its own limitations, we deduce that this conclusions is not largely changed even in the non-perturbative matching.

We thank all the member of the RBC and UKQCD Collaborations.

References

[1] P. Boyle [RBC and UKQCD Collaborations], PoS **LAT2007**, 005 (2007) [arXiv:0710.5880 [hep-lat]].

[2] E. Eichten and B. R. Hill, Phys. Lett. B **234**, 511 (1990).

[3] M. Della Morte, A. Shindler and R. Sommer, JHEP **0508**, 051 (2005) [arXiv:hep-lat/0506008].

[4] J. Heitger and R. Sommer [ALPHA Collaboration], JHEP **0402**, 022 (2004) [arXiv:hep-lat/0310035].

[5] O. Loktik and T. Izubuchi, Phys. Rev. D **75**, 034504 (2007) [arXiv:hep-lat/0612022].

[6] C. Albertus *et al.* [RBC and UKQCD Collaborations], PoS **LAT2007**, 376 (2007).

[7] N. H. Christ, T. T. Dumitrescu, O. Loktik and T. Izubuchi, PoS **LAT2007**, 351 (2007) [arXiv:0710.5283 [hep-lat]].

[8] E. Eichten and B. R. Hill, Phys. Lett. B **240**, 193 (1990).

[9] M. Albanese *et al.* [APE Collaboration], Phys. Lett. B **192**, 163 (1987).

[10] A. Hasenfratz and F. Knechtli, Phys. Rev. D **64**, 034504 (2001) [arXiv:hep-lat/0103029].

[11] C. J. Morningstar and J. Shigemitsu, Phys. Rev. D **57**, 6741 (1998) [arXiv:hep-lat/9712016].

[12] K. I. Ishikawa, T. Onogi and N. Yamada, Phys. Rev. D **60**, 034501 (1999) [arXiv:hep-lat/9812007].

[13] D. Becirevic and J. Reyes, Nucl. Phys. Proc. Suppl. **129**, 435 (2004) [arXiv:hep-lat/0309131].

[14] S. Aoki, T. Izubuchi, Y. Kuramashi and Y. Taniguchi, Phys. Rev. D **67**, 094502 (2003) [arXiv:hep-lat/0206013].